2.5 Logarithms

Positive real numbers: x, y, a, c, k
Natural number: n

104. Definition of Logarithm
$y = \log_a x$ if and only if $x = a^y$, $a > 0$, $a \neq 1$.

105. $\log_a 1 = 0$

106. $\log_a a = 1$

107. $\log_a 0 = \begin{cases} -\infty & \text{if } a > 1 \\ +\infty & \text{if } a < 1 \end{cases}$

108. $\log_a (xy) = \log_a x + \log_a y$

109. $\log_a \frac{x}{y} = \log_a x - \log_a y$

110. $\log_a (x^n) = n \log_a x$

111. $\log_a \sqrt[n]{x} = \frac{1}{n} \log_a x$

112. $\log_a x = \frac{\log_c x}{\log_c a} = \log_c x \cdot \log_a c$, $c > 0$, $c \neq 1$.

113. $\log_a c = \frac{1}{\log_c a}$

114. $x = a^{\log_a x}$

115. Logarithm to Base 10

$\log_{10} x = \log x$

116. Natural Logarithm

$\log_e x = \ln x$,

where $e = \lim_{k \to \infty} \left(1 + \frac{1}{k}\right)^k = 2.718281828\ldots$

117. $\log x = \frac{1}{\ln 10} \ln x = 0.434294 \ln x$

118. $\ln x = \frac{1}{\log e} \log x = 2.302585 \log x$